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Optimum Size of Plots in Multivariate Case

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Summary

The procedure to determine optimum size of experimental units was extended to the multivariate case. The matrix of relative dispersion was defined and its determinant was used as the measure of variation. Size of experimental units was optimised on three different considerations. The procedure was illustrated using data on cocoa.
Keyword: Maximum curvature, Dispersion matrix; Efficiency

Introduction

The success of any field experiment very much depends on the amount of experimental error which is a function of very many factors. The size and constitution of the experimental unit is a major factor contributing to the experimental error. Hence attention of researchers has been laid on determination of size and constitution of the experimental units so as to minimize the experimental error within the available resources. All attempts in the methodology as well as its application to various crops have so far been solely based on a single important character. But any crop is characterised by many characters and all of them have to be considered while studying it. In other words, it will be more meaningful to determine the optimum size of experimental unit based on simultaneous consideration of the various important characters of the crop.

2. *Measure of variation in Multivariate case*

Determinant of the dispersion matrix is in wide use as a measure of variation in multivariate case. But as in the univariate case it depends on the unit of measurement as well as the magnitude of the observations. Hence the matrix of relative dispersion of the vector variable

$$X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ . \\ . \\ X_p \end{bmatrix} \qquad (1)$$

was defined as $S = (S_{ij})_{p \times p}$ (2)

$$\text{where } S_{ij} = \sum_{k=1}^{N} \frac{X_{ik} X_{jk} - N\overline{X}_i \overline{X}_j}{N \overline{X}_i \overline{X}_j} \qquad i, j = 1, 2, \ldots, p,$$

x_{ik} is the observation on i^{th} character of the k^{th} unit, $\overline{X}_i$ is the mean per unit of the i^{th} character (X_i) and N is the total number of units.

Thus determinant of S ($|S|$) which is independent of units of measurement and magnitude of observations was proposed as the measure of relative variation in multivariate case for determination of optimum size of experimental unit.

3. *Determination of optimum plot size*

Three different approaches to determine optimum size of plot are discussed below.

1. Optimum plot size by the method of maximum curvature

Method of maximum curvature is in wide use in univariate case. Smith [4] empirically fitted a model.

$$\text{i.e.,} \quad V_x = V_1 x^{-b} \qquad (3)$$

where V_x is the variance of the yield per unit area for plots of x units and b, the index of soil heterogeneity. A modified form of this model, using coefficient of variation (CV) of units of size x in place of variance per unit area for plots of x unit is now in wide use (Hatheway and Williams [2], Agarwal [1]). This can be adopted in multivariate case also and thus model (3) can be fitted for $|S|$ against the plot size.

$$\text{i.e.} \quad Y = ax^{-b} \qquad (4)$$

where Y is the determinant of S for plots of x units. Maximising the curvature of (4), the optimum plot size can be arrived at as

$$X_{opt} = \{(ab)^2(2b+1)/(b+1)\}^{1/2(b+1)} \tag{5}$$

2. Plot size for achieving P% error

$$\text{Let} \qquad \overline{X} = \begin{bmatrix} \overline{X}_1 \\ \overline{X}_2 \\ \overline{X}_3 \\ \cdot \\ \cdot \\ \overline{X}_p \end{bmatrix} \tag{6}$$

be the mean vector for the p dimensional vector variable X for 'r' plots. The relative dispersion matrix or $\overline{X}$, say $D(\overline{X})$ is given by

$$D(\overline{X}) = \frac{S_{ij}}{r} \tag{7}$$

Its determinant is given by

$$|D(\overline{X})| = \frac{|S|}{r^p}$$

Analogous to fixing CV at P% level in univariate case, for P% error in multivariate case,

$$\frac{|S|}{r^p} = \left(\frac{p}{100}\right)^{2p}$$

$$\text{i.e.,} \; \frac{|S|^{1/p}}{\left(\frac{p}{100}\right)^2} = r \tag{8}$$

is the number of replications required to achieve P% error. In other words, the number of replications, r, achieve P% error has to be at least $|S|^{1/p}/(p/100)^2$. The experimental area required to achieve P% error may be obtained by multiplying the number of replication with the corresponding plot size. Select that plot size which require minimum experimental area as the optimum.

3. Plot size having maximum efficiency

Kalamkar [3] defined efficiency of plots of x units as $1/xC_x$ in univariate case, where C_x is the coefficient of variation of plots of x units. Efficiency of a plot of x units in multivariate case can be defined as

$$\frac{1}{x} \sqrt[p]{|S|} \tag{9}$$

The plot size which gives maximum efficiency can be taken as the optimum.

4. *Illustration*

Observations on three characters viz., yield, canopy spread and girth at 15cm height for 738 trees of 'forastero' variety of cocoa (*Theobroma cacao* L.) maintained in Kerala Agricultural University Farm, Trichur, have been used for the purpose of illustration. Single tree plot was taken as the smallest unit. Determinant of |S| was calculated for plots of size ranging from one to fifteen by combining adjacent plots and are provided in Table 1.

Table 1. |S|, No. of replications, trees required to achieve 5% error and efficiency for different sizes of plots

Plot size	$\lvert S\rvert \times 10^5$	No. of Replications for 5% error	No. of trees for 5% error	$\frac{1}{x}\sqrt[3]{\lvert S\rvert}$
1	47.477	29	29	12.86
2	8.295	17	34	11.51
3	3.110	13	39	10.61
4	1.768	10	40	9.72
5	1.019	9	45	9.28
6	0.738	8	48	8.59
7	0.453	7	49	8.65
8	0.313	6	48	8.57
9	0.272	6	54	7.97
10	0.249	5	50	7.46
11	0.198	5	55	7.34
12	0.147	4	48	7.45
13	0.130	4	52	7.05
14	0.119	4	56	6.92
15	0.082	4	60	7.12

Model (4) was estimated to be $Y=0.000410x^{2.26}$. By the method of maximum curvature, optimum plot size was obtained using (5) as single tree plots.

Number of replications and number of trees required to achieve 5% error are also presented in Table 1. On examination of the Table, single tree plots were found to require least area and hence the optimum plot size.

Using equation (8) efficiency for plots of different sizes were obtained and are given in Table 1. It may be noted that single tree plots have the maximum efficiency and hence the optimum size of plot.

Thus single tree plots were found to be optimum on all the three considerations.

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